The Economics of Asset Management[1]

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ABSTRACT

This paper develops a contingent claims approach to valuing the fee flow generated by mutual funds. The method incorporates the key empirical characteristic of mutual funds, namely the relation between fund performance and fund inflows. Using this approach, we address a number of central issues in asset management such as the risk-taking incentives of mutual funds, the value of active management, and the optimal fund contract.

1 Introduction

Mutual funds represent one of the more important financial intermediaries in the economy. At the end of 2002, the mutual fund industry managed over $11 trillion dollars of assets worldwide according to data from the Investment Company Institute. Given the size of this industry, it is perhaps not surprising that a considerable academic literature has emerged. This literature has studied extensively the performance of mutual funds, how fund flows respond to this performance, the equilibrium nature of active management, and the principal-agent conditions inherent in the industry.[1] This paper adds to this literature by answering the fundamental question what determines the economic value of a mutual fund? Understanding mutual fund valuation is a necessary building block to addressing the central issues of asset management such as the risk-taking incentives of mutual funds, the value of active management, and the optimal fund contract.

Specifically, this paper develops a valuation methodology for the fee flow generated by mutual funds ("mutual fund valuation" henceforth). We employ a contingent claims approach to mutual fund valuation by recognizing that mutual funds have the essential characteristics of asset-backed securities. The key insight underlying our valuation approach is that in an efficient market the present value of a claim on the future value of an asset is simply a function of the current value of that asset. The result is that the value can be written in terms of current asset values and a number of observable parameters. In particular, the paper incorporates the key empirical characteristic of mutual funds, namely the relation between fund performance and fund inflows.[2]

This paper complements Goetzmann, Ingersoll and Ross (2003) who value hedge fund management contracts under incentive fees and high-water mark provisions. While their paper focuses on features that are specific to the hedge fund industry, one could infer mutual fund values that are similar to some special cases of the model provided below by assuming no incentive fees and no high-water marks. With respect to hedge funds, Goetzmann, Ingersoll and Ross (2003) also discuss evidence that the flow of capital into the fund (and thus the growth rate of the assets) depends on changes in the fund's value and other characteristics of the fund. However, they do not build this complicating feature into the valuation framework per se. Moreover, they argue that the incentive fee structure limits the amount of assets going into the fund. The issue of fund flows and performance is key from a valuation perspective

[1]See, for example, respectively Jensen (1998), Malkiel (1995), and Carhart (1997); Ippolito (1992), Sirri and Tufano (1998) and Bergstrasser and Poterba (2002); Gruber (1996), Nanda, Narayaran and Warther (2000), Lynch and Musto (2004) and Berk and Green (2004); and Brown, Harlow and Starks (1996), Chevalier and Ellison (1997) and Das and Sundaram (2002), among others.

[2]Independently, Basak, Pavlova and Shapiro (2004) develop a model of mutual fund manager behavior in the presence of the relation between fund flow and performance.

and highly relevant for the mutual fund industry, and is the focus of our paper.

The paper provides two primary contributions to the economics of asset management. The first is the valuation methodology for mutual funds. As well as providing a closed-form solution for the value (albeit in a simple setting), the paper also presents new empirical estimates of the fund flow-performance relation and then incorporates these estimates into the valuation. The second is to use mutual fund valuation to shed light on a number of the issues described above. As a flavor of the analysis, consider the question of how can one understand a mutual fund company's incentives in its role as a financial intermediary without the broader context of how mutual funds are actually valued? For example, it is taken as given that the nonlinear relation between fund flow and performance provides the incentives for risk taking by mutual funds (e.g., Brown, Harlow and Starks (1996), Chevalier and Ellison (1997), Sirri and Tufano (1998) and Das and Sundaram (2002)). The paper shows that nonlinearity is not a necessary condition for risk taking and, in fact, under certain conditions, the mutual fund company will actually reduce risk by more closely tracking the benchmark.

The paper is organized as follows. Section 2 presents a closed-form solution (and corresponding analysis) for valuing mutual funds when fund flows depend on changes in the underlying fund's net asset value (NAV). In Section 3, we present an empirical fund flow model for mutual funds. In Section 4, we apply the valuation approach under the empirical fund flow specification. The results imply an optimal tracking error variance and fee policy for mutual funds. Moreover, some general implications about mutual fund values and risk are derived. Section 5 makes some concluding remarks and suggests directions for future research.

2 A Closed-form Solution to Mutual Fund Valuation

Throughout this paper we assume that mutual funds have zero alphas, that is, fund managers do not possess superior skill at choosing undervalued assets. We make this assumption in order to avoid the debate about the presence of excess performance or lack thereof in the mutual fund industry.[3] In theory, a constant alpha would be straightforward to incorporate.

As with the usual Black-Scholes framework, we assume that the net asset value of the fund, S_t, follows a lognormal diffusion process. The assumption of a constant, known volatility seems especially strong as the fund manager's trading strategy may lead to an unknown (and possibly stochastic) variance of the fund return. We recognize this is a strong assump-

[3] See, for example, Ippolito (1992), Elton, Gruber and Blake (1996), and Carhart (1997) for differing evidence, and Berk and Green (2004) for a recent theoretical paper addressing this issue.

tion, but similar to Goetzmann, Ingersoll and Ross (2003) we abstract from it in this section of the paper. In later sections, we describe the optimal strategy for choosing volatility.

Ceteris paribus, the NAV of the fund will decline because of fees paid to the manager of the fund company as well as cash distributions (i.e., dividends and capital gains) to investors. With respect to fees, we assume that the fee is paid continuously at (annualized) rate c. While in practice fees are generally taken out at discrete points, the fee as a constant proportion of assets is typical across all mutual funds. In this paper, we abstract from the issue of distributions.[4]

The focus of this section is to characterize in a reasonable setting the relation between the fund's value and the underlying assets under management when flows depend on the fund's performance. Write S_t for the value of a mutual fund share, and I_t for the value of the underlying benchmark index. Assume their joint dynamics can be written as

$$\begin{aligned} dI_t &= \mu_I I_t\, dt + \sigma_I I_t\, dZ_I, \\ dS_t &= \mu_S S_t\, dt + \sigma_S S_t\, dZ_S, \\ dZ_S\, dZ_I &= \rho\, dt, \end{aligned}$$

where the correlation between dZ_I and dZ_S is ρ_{IS}. Also suppose that a representative agent exists, who holds the index. Any asset can be priced using his pricing kernel,[5]

$$d\pi_t/\pi_t = -r\, dt + \left[\left(\mu_I - r\right)/\sigma_I\right]\, dZ_I.$$

Write $f(S, I)$ for the value of an arbitrary claim whose value depends on I_t and S_t, and which has a payout rate d_t. By the properties of a pricing kernel, and using Ito's Lemma, we have

$$\begin{aligned} 0 &= E[d(\pi_t f)] + d_t\, dt, \\ &= E\left[\pi_t\, df + f\, d\pi_t + df\, d\pi_t + d_t\, dt\right], \\ &= \pi_t \left(f_t + f_S \mu_S S + f_I \mu_I I + \frac{1}{2} f_{SS} \sigma_S^2 S^2 + \frac{1}{2} f_{II} \sigma_I^2 I^2 + f_I S \sigma_I \sigma_S \rho_{IS} I S \right) dt + \\ &\qquad f\left(-\pi_t r\right) dt - f_S \sigma_S S \frac{(\mu_I - r)}{\sigma_I} \rho_{IS} \pi_t\, dt - f_I \sigma_I I \frac{(\mu_I - r)}{\sigma_I} \pi_t\, dt + d_t\, dt. \end{aligned}$$

[4]Note that most distributions are reinvested in the funds, so that the effect on the underlying assets under management is small. That is, the NAV decline is offset by the increase in the number of shares of the fund.

[5]We do not derive pricing relations using no-arbitrage arguments, as in Black and Scholes (1976). Instead, we use an equilibrium derivation similar to Goetzmann, Ingersoll and Ross (2003, pp. 1689–1690) [see also Merton (1976) and Ingersoll (2002).]

Assume that

$$\mu_S = r + \beta_S(\mu_I - r) - c,$$

where $\beta_S = \sigma_S \sigma_I \rho_{IS}$, the CAPM beta of S, and c is the annual management fee.[6] Then this equation simplifies to

$$0 = f_t + f_S(r-c)S + f_I r I + \frac{1}{2} f_{SS}\sigma_S^2 S^2 + \frac{1}{2} f_{II}\sigma_I^2 I^2 + f_I S \sigma_I \sigma_S \rho_{IS} I S - rf + d_t.$$

By the Feynman-Kac formula (see, for example, Duffie (1988)), the solution to this equation is

$$f = E_t\left[e^{-rT-t} f(\widehat{S}_T, \widehat{I}_T) + \int_t^T e^{-r(s-t)} d(\widehat{S}_s, \widehat{I}_s, s)\, ds\right], \tag{1}$$

where

$$d\widehat{I}_t = r\widehat{I}_t\, dt + \sigma_I \widehat{I}_t\, dZ_I, \tag{2}$$

$$d\widehat{S}_t = (r-c)\widehat{S}_t\, dt + \sigma_S \widehat{S}_t\, dZ_S, \tag{3}$$

$$dZ_S\, dZ_I = \rho\, dt, \tag{4}$$

$$\widehat{I}_t = I_t, \tag{5}$$

$$\widehat{S}_t = S_t. \tag{6}$$

In other words, we can use the usual Black-Scholes type result that an asset's value equals its expected discounted payoff in a world where assets follow the risk-adjusted processes given in Equations (2)–(3).

Let the number of shares in the fund be N_t, so that the total value of assets under management in the fund is

$$V_t = N_t S_t.$$

The goal of our paper is to better understand the interaction between N and S in light of the existing empirical evidence and the effect of this interaction on fund valuation. We treat the flow of monies into and out of the fund as a lognormal diffusion process governed by, among other factors, a constant mean ν and an independent volatility shock σ_N. As our empirical analysis shows there is an important fraction of flow variations which is unexplained. As long

[6] We assume that the mutual fund earns expected returns less than the underlying assets due to the fees paid to the mutual fund company. However, as long as the mutual fund cannot be shorted, there is no possibility of arbitrage. There does exist the important issue of why individuals invest in mutual funds in the first place under these circumstances. One possibility is that investors face lower transactions costs, either explicitly through trading constraints or implicitly through time constraints. Alternatively, mutual fund managers may possess superior investment skills, i.e., positive alpha.

as this variation is not correlated with the underlying NAV of the fund and can be diversified away within a broader set of asset holdings, this noise will have little impact on the fund's value and priced risk (though not its idiosyncratic risk). Nevertheless, the assumption that net flows depend on the change in the underlying NAV is clearly the most important piece of evidence and we incorporate this stylized fact below. In particular, we assume that the flow of monies into and out of the fund depends on the contemporaneous return of the NAV of the fund relative to some benchmark. We specify this dependence as a linear relation:

$$
\begin{aligned}
dN_t/N_t &= \nu\, dt + \theta\left([dS_t/S_t - r] - \gamma[dI_t/I_t - r]\right) + \sigma_N\, dZ_N, && (7)\\
&= (\nu - \theta c)\, dt + \theta\left(\sigma\, dZ_S - \gamma\sigma_I\, dZ_I\right) + \sigma_N\, dZ_N. && (8)\\
dZ_N\, dZ_S &= dZ_N\, dZ_I = 0. && (9)
\end{aligned}
$$

Several observations are in order. First, the parameter γ describes the rule investors use to judge the fund's performance versus the benchmark index. For example, $\gamma = 1$ compares the return on the fund's NAV to the index's return without adjusting for relative risk (see Sirri and Tufano (1998), Chevalier and Ellison (1997) and Bergstresser and Poterba (2002)); $\gamma = 0$ is equivalent to investors looking at absolute rather than relative performance; and γ equal to the fund's beta, i.e. $\frac{\mathrm{cov}(dS_t/S_t, dI_t/I_t)}{\mathrm{var}(dI_t/I_t)}$ adjusts for risk (see Gruber (1996), Del Guercio and Tkac (2002), and most performance measurement studies). In this latter case, $\sigma\, dZ_S - \gamma\sigma_I\, dZ_I$ in equation (7) can be identified as the well-known tracking error of the fund.

Second, the model above assumes a linear relation between fund flows and performance. This is in stark contrast to the literature which tends to find a convex relation. We make this assumption for tractability sake for the closed-form solution. In later sections, we use the actual empirical relation and therefore relax this assumption. As a preview of our findings, while the empirical relation matters greatly, the convexity versus linearity question does not alter the main finding that the fund's value depends on the underlying volatility of the assets under management.

Third, by linking flows to net (and not gross) returns on the fund, there is a tendency for the net flows to decrease due to the negative effect of fees on NAVs. Of course, in practice, these fees may be related to differing alphas across funds.[7] As described earlier in this section, we are assuming that the managers of the funds do not have the ability to generate excess performance. In addition, there is considerable evidence linking both ν and θ to the size of the fund, the type of fund, the fee structure, the fund's age, etc. These features will be built into our empirical model of fund flows in the next section.

[7]However, the evidence in the literature tends to support the opposite conclusion (e.g., Gruber (1996)).

Fourth, one of the main stylized facts from the empirical literature is that there is a lag between the realization of fund performance and its effect on fund flows, one year being a typical lag chosen in this literature (e.g., Sirri and Tufano (1998)). In the continuous-time framework above (though not in later sections), we abstract from this feature and assume the effect is instantaneous. This is not an important distinction for valuation. Since the fund's value depends on the present value of all future fees, and these fees depend on the assets and net flows, whether the flow occurs this period or next period is a second-order effect.

Note that one can view the valuation to follow as being from the perspective of an investor in a competitive market who wishes to know what the value of this fund's particular revenue stream is. Since the investment management company may be a profit maximizing entity of behalf of its investors, the valuation can be treated from the management company's perspective. This valuation, however, may not align necessarily with the individual portfolio manager though in theory the company may be able to write a contract in such a way as to ensure the alignment of incentives. This paper abstracts from this particular agency problem.

Define $s_t \equiv \log S_t$, $i_t \equiv \log I_t$, and $n_t \equiv \log N_t$. By Ito's Lemma,

$$ds_t = \left(r - c - \frac{1}{2}\sigma^2\right) dt + \sigma \, dZ_S, \tag{10}$$

$$di_t = \left(r - \frac{1}{2}\sigma_I^2\right) dt + \sigma_I \, dZ_I, \tag{11}$$

$$\begin{aligned} dn_t = {} & \left(\nu - \theta c - \frac{1}{2}\theta^2 \left[\sigma^2 + \gamma^2\sigma_I^2 - 2\rho\sigma\gamma\sigma_I\right] - \frac{1}{2}\sigma_N^2\right) dt \\ & + \theta \left(\sigma \, dZ_S - \gamma\sigma_I \, dZ_I\right) + \sigma_N \, dZ_N. \end{aligned} \tag{12}$$

Integrating these equations, we obtain

$$s_\tau = s_t + \left(r - c - \frac{1}{2}\sigma^2\right)(\tau - t) + \sigma \int_t^\tau dZ_S. \tag{13}$$

$$i_\tau = i_t + \left(r - \frac{1}{2}\sigma_I^2\right)(\tau - t) + \sigma_I \int_t^\tau dZ_I. \tag{14}$$

$$\begin{aligned} n_\tau = {} & n_t + \left(\nu - \theta c - \frac{1}{2}\theta^2 \left[\sigma^2 + \gamma^2\sigma_I^2 - 2\rho\sigma\gamma\sigma_I\right] - \frac{1}{2}\sigma_N^2\right)(\tau - t) \\ & + \theta\sigma \int_t^\tau dZ_S + \theta\gamma\sigma_I \int_t^\tau dZ_I + \sigma_N \int_t^\tau dZ_N. \end{aligned} \tag{15}$$

From equations (13) – (15), s_τ, i_τ and n_τ are normally distributed for all τ.

Write $M_{t,T}$ for the present value of all fees between t and some terminal horizon, T (which might be infinite). Then

$$M_{t,T} = c\,E_t \int_t^T e^{-r(\tau-t)} V_\tau \, d\tau, \tag{16}$$

or, switching the integral and the expectation,

$$M_{t,T} = c \int_t^T E_t \left[e^{-r(\tau-t)} V_\tau \right] \, d\tau. \tag{17}$$

Since both n_t and s_t are normal, so is their sum, and

$$\begin{aligned} E_t \left[e^{-r(\tau-t)} V_\tau \right] &= E_t \left[e^{n_\tau + s_\tau - r(\tau-t)} \right], \\ &= \exp \left[E_t(n_\tau + s_\tau) + 1/2 \, \mathrm{var}_t(n_\tau + s_\tau) - r(\tau - t) \right]. \end{aligned} \tag{18}$$

From equations (13) and (15), we have

$$\begin{aligned} E_t(s_\tau + n_\tau) &= E_t(s_\tau) + E_t(n_\tau), \\ &= s_t + n_t + \\ &\quad \Bigg(r + \nu - c(1+\theta) - \frac{1}{2}\sigma^2 \\ &\qquad - \frac{1}{2}\theta^2 \left[\sigma^2 + \gamma^2 \sigma_I^2 - 2\rho\sigma\gamma\sigma_I \right] - \frac{1}{2}\sigma_N^2 \Bigg) (\tau - t). \end{aligned} \tag{19}$$

$$\begin{aligned} \mathrm{var}_t(s_\tau + n_\tau) &= \mathrm{var}_t(s_\tau) + \mathrm{var}_t(n_\tau) + 2\,\mathrm{cov}_t(n_\tau, s_\tau), \\ &= \sigma^2(\tau - t) + \left\{ \theta^2 \left[\sigma^2 + \gamma^2 \sigma_I^2 - 2\rho\sigma\gamma\sigma_I \right] + \sigma_N^2 \right\} (\tau - t) \\ &\qquad + 2 \left(\theta\sigma^2 - \theta\rho\sigma\gamma\sigma_I \right) (\tau - t). \end{aligned} \tag{20}$$

Substituting into equation (18) and simplifying, we obtain

$$E_t \left[e^{-r(\tau-t)} V_\tau \right] = V_t \, e^{-[c(1+\theta) - \nu - \theta\sigma(\sigma - \rho\gamma\sigma_I)](\tau - t)}. \tag{21}$$

Finally, performing the integration in equation (17), we obtain[8]

$$\begin{aligned} M_{t,T} &= cV_t \int_t^T e^{-[c(1+\theta) - \nu - \theta\sigma(\sigma - \rho\gamma\sigma_I)](\tau - t)} \, d\tau, \\ &= \frac{c \left[1 - e^{-[c(1+\theta) - \nu - \theta\Sigma](T-t)} \right)}{c(1+\theta) - \nu - \theta\Sigma} V_t, \end{aligned} \tag{22}$$

[8] As long as $c(1+\theta) \neq \nu + \theta\sigma(\sigma - \rho\gamma\sigma_I)$. Otherwise, $M_{t,T} = c\,V_t(T - t)$.

where $\Sigma \equiv \sigma(\sigma - \rho\gamma\sigma_I)$. Letting the terminal horizon go to infinity, we have

$$M_t \equiv \lim_{T\to\infty} M_{t,T} = \frac{c}{c(1+\theta) - \nu - \theta\Sigma} V_t \quad \text{for } c(1+\theta) > \nu + \theta\Sigma. \tag{23}$$

Equations (22) and (23) provide the intuition for understanding the relation between mutual fund values and the value of the underlying assets under management.[9] As a first pass at the intuition, consider the case in which fund flows are unrelated to fund performance, i.e., $\theta = 0$. Substituting $\theta = 0$ into equation (23), we get

$$M_t = \frac{c}{c - \nu} V_t \quad \text{for } c > \nu. \tag{24}$$

Equation (24) is essentially the Gordon growth model in which the initial cash flow is the fee earned on the assets under management, i.e., cV_t. Define the other parameters of the Gordon growth model as r^* for the discount rate and g for the growth rate. For this case, $g = r^* - c + \nu$. Since the assets are expected to grow at their expected return, adjusted for fees and exogenous growth, the expected return cancels. This is one of the central ideas of the paper, and for that matter any contingent claim analysis – the present value of the future asset price is just today's price. The present value of the fees then depends on the remaining two components of the growth rate. The fee, c, leads to a lower growth rate as the NAVs decline in c, while ν represents general growth in net flows not related to the NAV.

The more interesting case, and the primary contribution of the paper, is to consider $\theta \neq 0$. The valuation formula in equation (23) now has two additional terms affecting the growth rate of the cash flows. The first term is $c\theta$. θ measures the sensitivity of the net flows to the fund's NAV return relative to the benchmark and is most likely positive. Therefore, because fees force NAVs to drift downward through time, $c\theta$ hurts the cash flow growth. The second term is $\theta\Sigma$. Σ has a particularly interesting economic interpretation. It represents the variance of the tracking error between the fund and the benchmark, or, in statistical terms, $\sigma^2(1 - R^2)$, where R^2 is the regression r-squared of the fund return on the benchmark return. Assuming again that $\theta > 0$, Σ increases the fund's value as idiosyncratic shocks to the fund's return will affect both NAVs and fund flows. This is the convexity effect that is common to option pricing and other asset-backed security pricing, such as mortgage-backed securities. (See also Carpenter (2000) and Basak, Pavlova and Shapiro (2004).)

[9] The infinite horizon case is not far-fetched. Unlike hedge funds, mutual funds tend not to disappear. This is partly due to the lack of a high watermark, but also to the fact that poorly performing funds are generally merged into existing funds (Gruber (1996)). Thus, a long horizon setting may be appropriate for analyzing the present value of fees.

2.1 Risk-taking Incentives of Mutual Funds

In order to understand the risk-taking incentives of mutual funds, it seems reasonable to consider mutual fund valuation as a starting point. There is an extensive literature that looks at the incentives of fund managers without this first step, e.g., Grinblatt and Titman (1989), Brown, Harlow and Starks (1996), Chevalier and Ellison (1997), Carpenter (2000), Busse (2001), Das and Sundaram (2002), and Golek and Starks (2004). Of some interest, Brown, Harlow and Starks (1996) and Chevalier and Ellison (1997) empirically analyze the incentives of mutual fund managers to increase the flows into the fund. Under the assumption that flows go more quickly into the very best performing funds (yielding an option-like payoff), or under the assumption that investors look at year-end returns so that poor performers at mid-year have different incentives, they argue and show empirically that managers may take more volatile positions.[10] As previously mentioned, Basak, Pavlova and Shapiro (2003) can be considered as a formal model of these papers.

In our framework, we also find a role for incentives in that we show the value of the fund increases as a function of tracking error variance, Σ (as long as $\theta > 0$). The incentive of mutual fund managers is to therefore choose assets which are both volatile and less correlated with the benchmark index. Of course, the ability to take such actions are subject to the constraints implied by the contract between investors and the fund. At first glance, the effect of Σ described above confirms the previous analyses. Note that the result in equation (23), however, shows that neither the relative ranking nor nonlinearity in the flow-performance relation are important per se for the managerial incentive to take greater risk. This incentive arises quite naturally from the nonlinear payoff structure. This is true even if the flow-performance relation is linear (as in equation (7)). Moreover, our finding contrasts with Goetzmann, Ingersoll and Ross (2003) who also document a similar effect of increasing the fund's volatility. Their result derives from the incentive fee in which hedge funds take a proportion of the return above the high-water mark. Here, even with no incentive fee, this result carries through because fund flow depends on the fund's performance.

Nevertheless, the empirical results of Chevalier and Ellison (1997), as well some of the previously cited literature, are important. They find that θ varies across fund characteristics as well as across fund performance. Since the effect of tracking error volatility on the mutual fund's value depends very much on the sign and magnitude of θ, the empirical properties of θ will be important economically. The intuition is that the greater the sensitivity the greater the convexity of the payoff between the mutual fund's value and the underlying NAV. A primary focus of our empirical analysis in Section 3 is to characterize the fund's θ as a

[10] The empirical evidence in support of these claims is not universally accepted (e.g., Busse (2001)).

function of the characteristics and performance of the fund.

2.2 The Value of Active Management

How can we address the central issue of why active management exists without a deep understanding of how much wealth is transferred from investors to the various constituents of the mutual fund over its life (e.g., shareholders, employees, leaseholders, marketing agents, etc)? The valuation equations (22) and (23) provide a direct estimate of this transfer. As an example, consider the aforementioned $\theta = 0$ case at an infinite horizon. If we assume no growth in fund flows (i.e., $\nu = 0$), then the fund's value equals the current assets under management, V_t. In other words, in present value terms, the management company extracts all the value of the assets. This is analogous to the house eventually owning the pot in a poker game by taking a small cut of each hand played. The idea that wall street businesses produces substantial revenue streams in a present value sense is not new, e.g., see Fred Schwed's famous 1940 book *Where are the Customer's Yachts?*.

Of course, investors into the fund come and go, so the above transfer refers to the aggregate wealth "donated" by the investors in the fund throughout its life. In equilibrium, these transfers should be offset by the services provided by the mutual fund. For example, in this paper, we assume a zero alpha. A positive alpha could recoup all of these transfers (e.g., Berk and Green (2004)). Even in the presence of a zero alpha, these transfers may be much smaller than implied by the infinite horizon model. In fact, in later sections, we show that this is the case when we provide direct estimates of this transfer as a function of estimated parameters and assumptions about the fund's age, size and fee characteristics.

In order to develop some intuition, however, one can compare the infinite and finite horizon cases by deriving the speed at which this asset transfer takes place. In particular, by comparing equations (22) and (23), it is immediate that

$$M_{t,T} = M_t \left(1 - e^{-[c(1+\theta)-\nu-\theta\Sigma](T-t)}\right). \tag{25}$$

The period t finite horizon value of a fund that will die at $t+T$ is a fraction of its infinite value. To garner some intuition we derive the fund's half-life – the time it takes for the fund to accumulate half of its infinite value. Defining the half life horizon to be $T^* - t$ we get

$$T^* - t = \frac{\ln(2)}{c(1+\theta) - \nu - \theta\Sigma}. \tag{26}$$

The half-life falls in the fee, c, as raising fees increases the rate at which managers' take their cut; rises in convexity-adjusted fund flow growth, $\nu + \theta\Sigma$ as growth increases future values;

and is ambiguous in θ as the fee effect battles it out with the convexity effect. Consider the case, $\theta = 0$. Mutual funds are prevented from owning all the assets in a present value sense by the length of the finite horizon or by declining growth rates in flows as the fund's size increases. (This latter feature will be built into our model of fund flow valuation in Sections 3 and 4.) Now assume $\nu = 0$. In terms of the half life calculations, equation (25) reduces to $T^* - t = \frac{1}{c} \ln(2)$. With an annualized fee of 2% one half of the funds will go to management fees in 35 years, while for a fund with one tenth of the fees, half the value will be obtained in 347 years.

2.3 The Optimal Fee Contract

While there is a large literature on optimal mutual fund contracts (e.g., Chordia (1996), Admati and Pfleiderer (1997), Das and Sundaram (2002) and Christoffersen (2003), among others), none of these papers incorporate the present value effect of fees. This may be problematic as the valuation equation (23) provides surprising results on how the fund's value changes relative to the fees it charges. If, for example, $\nu + \theta\Sigma > 0$, which can be thought of as a convexity-adjusted growth rate in flows, then the mutual fund value actually falls as the manager increases the fee. While higher fees increase current revenue, they tend to decrease the net asset value of the fund over time via its negative impact on asset growth. In a present value sense, it would be better to let the flows grow quickly and reap the present value of all future fees than take higher fees upfront.

Of course, the infinite horizon example assumes these parameters last forever. Consider the finite horizon value given by equation (22). For given values of V_t, θ, ν, Σ, t and T, we can show that there is a unique value of $c > 0$ that maximizes $M_{t,T}$, provided that $\nu + \theta\Sigma > 0$. The intuition is that setting c too low is bad for managers, as the fund terminates before they have taken much money out in the form of fees. On the other hand, setting c too high is also bad, as it means the fund will shrink in future as investors withdraw their money in response to poor net performance. The optimal c balances these two offsetting considerations.

Suppose now that the manager is free to change the fee over time, so we can write the dynamics of S_t as[11]

$$dS_t = (r - c(t))\ S_t\, dt + \sigma S_t\, dZ_S, \tag{27}$$

The literature, and corresponding empirical analysis in Section 3, show that the growth rate, ν, is decreasing while the sensitivity to performance, θ, is increasing in the fee. As c is a control variable, it is important to incorporate these stylized facts into the fund flow

[11]Homogeneity of the problem implies that the optimal c may be a function of time, but will *not* depend on S_t or V_t.

dynamics, namely

$$
\begin{aligned}
dN_t/N_t &= (\nu - \psi c_t)\,dt + (\theta + \kappa c_t)\,([dS_t/S_t - r\,dt] - \gamma[dI_t/I_t - r\,dt]) + \sigma_N\,dZ_N, &(28)\\
&= (\nu - (\psi+\theta)c_t - \kappa c_t^2)\,dt + (\theta + \kappa c_t)\,(\sigma\,dZ_S - \gamma\sigma_I\,dZ_I) + \sigma_N\,dZ_N. &(29)\\
dZ_N\,dZ_S &= dZ_N\,dZ_I = 0, &(30)
\end{aligned}
$$

where ψ and κ measure c's effect on ν and θ, respectively. By Ito's Lemma, the dynamics of V are

$$
\begin{aligned}
dV_t/V_t &= \left[r + \nu + \theta\Sigma - Ac_t - \kappa c_t^2\right]\,dt + \\
&\qquad \sigma(1 + \theta + \kappa c_t)\,dZ_S - (\theta + \kappa c_t)\gamma\sigma_I\,dZ_I + \sigma_N\,dZ_N, \qquad (31)
\end{aligned}
$$

where $A = 1 + \theta + \psi - \kappa\Sigma$. Define $M^c(v,t) = E_{cx}\left[\int_t^T e^{-r(\tau-t)}c_\tau V_\tau\,d\tau\right]$, where $V_t = v$, and c is a possible fee schedule. Also, define $M(v,t) = \sup_{c\in C} M^c(v,t)$. By homogeneity, the solution must be of the form $M(v,t) = v\,f(t)$, for some function $f(t) \equiv M/V$.

The Bellman equation [Duffie (1988, page 270)] for the optimal choice of c can be written in the form

$$\sup_{c_t}\left\{f_t + f\left[\nu + \theta\Sigma - Ac_t - \kappa c_t^2\right] + c_t\right\} = 0. \qquad (32)$$

Differentiating with respect to c and setting to zero yields the solution for the optimal c in terms of f:

$$c^* = \frac{1 - Af(t)}{2\kappa f(t)}, \qquad (33)$$

and substituting this into equation (32) gives a differential equation for f:

$$f_t + f\left(\nu + \theta\Sigma + \frac{A^2}{4\kappa}\right) - \frac{A}{2\kappa} + \frac{1}{4\kappa f} = 0, \qquad (34)$$

with boundary condition

$$f(T) = 0.$$

In general this equation can only be solved numerically, but we can derive several results analytically. For very short horizons ($t \to T$),[12]

$$f(t) \approx \sqrt{\frac{T-t}{2\kappa}}, \tag{35}$$

$$c^* \approx \frac{1}{2\kappa}\left(\sqrt{\frac{2\kappa}{T-t}} - A\right). \tag{36}$$

For very long horizons ($T \gg t$),

$$c^* \approx \begin{cases} \sqrt{\frac{-\nu+\theta\Sigma}{\kappa}} & \text{If } \nu^* < 0, \\ \frac{-A}{2\kappa} & \text{If } \nu + \theta\Sigma \geq 0. \end{cases} \tag{37}$$

From equation (37), if A is positive and $\nu + \theta\Sigma \geq 0$, the optimal value of c is actually negative for long horizons. Intuitively, the manager may be better off forgoing compensation today (or even putting money into the fund) in exchange for a larger fund, and hence higher compensation, in the future. This result is consistent with Christoffersen (2001) who reports strong evidence that mutual funds (albeit from the money market asset class) waive fees. Moreover, these waivers are more likely for new funds. If one recognizes that mutual funds have a number of fixed expenses, one could make the argument that fee waiving can effectively be equivalent to choosing a negative c. To complete the picture, note that from equation (33),

$$\begin{aligned} \frac{\partial c^*}{\partial f} &= \frac{-1}{2kf^2}, \\ &\leq 0. \end{aligned}$$

In addition, it is a simple matter to show that[13] $\frac{df}{dt} \leq 0$. Combining these two results, we obtain

$$\begin{aligned} \frac{dc^*}{dt} &= \frac{dc^*}{df} \times \frac{df}{dt}, \\ &\geq 0. \end{aligned} \tag{38}$$

In other words, for a fund with a very long initial maturity date, the optimal compensation level starts close to the value given in equation (37), is always increasing over time, and

[12]This model for fees is highly stylized and is not meant to capture actual practice. The optimal strategy here is to charge very large fees as $t \to T$ in order to extract all the wealth from the fund that is about to be shut down. For both legal reasons and most probably market pressures, it is difficult for mutual funds to increase their fees throughout time. Thus, the fee, c, is bounded from above.

[13]Intuitively, for a given fund value, the manager is always better off with a longer remaining horizon.

approaches the expression given in equation (36) as the maturity date of the fund approaches.

2.4 The Return-Risk Profile of Asset Management

The field of finance often concerns itself with the distribution of returns, rather than prices, of assets. The valuation equations for mutual funds allow us to characterize the fundamental determinants of mutual fund business returns, and, in particular, their return and risk profile.

Specifically, first rewrite equation (12) as

$$\begin{aligned} dn_t &= \left[\nu - \frac{1}{2}\theta(\theta-1)\sigma^2 - \frac{1}{2}\theta\gamma(\theta\gamma+1)\sigma_I^2 + \theta^2\rho\sigma\gamma\sigma_I + \theta r(\gamma-1) - \frac{1}{2}\sigma_N^2\right] dt + \\ &\qquad \theta\, ds_t - \theta\gamma\, di_t + \sigma_N\, dZ_N. \end{aligned} \tag{39}$$

Integrating this and taking exponentials, we can write the number of shares of the fund at some future date, $N_{t+\tau}$, in terms of the size at time t and the growth in net asset value. Defining the return on the fund's NAV and the index as, respectively, $R^S_{t,t+\tau} \equiv \left[\frac{S_{t+\tau}}{S_t}\right]$, and $R^I_{t,t+\tau} \equiv \left[\frac{I_{t+\tau}}{I_t}\right]$, we obtain

$$\frac{N_{t+\tau}}{N_t} = e^{\left[\nu-\frac{1}{2}\theta(\theta-1)\sigma^2-\frac{1}{2}\theta\gamma(\theta\gamma+1)\sigma_I^2+\theta^2\rho\sigma\gamma\sigma_I+\theta r(\gamma-1)-\frac{1}{2}\sigma_N^2\right]\tau+\sigma_N\left(\tilde{Z}_{N,t+\tau}-\tilde{Z}_{N,t}\right)} \left(\frac{{R^S_{t,t+\tau}}^{\theta}}{{R^I_{t,t+\tau}}^{\gamma\theta}}\right), \tag{40}$$

where $\tilde{Z}_{N,t}$ represents the sum of all the idiosyncratic shocks to net flows. Because the net flows, dN_t, depend on both dS_t, dI_t and dZ_N, the number of shares in the future depends on both the underlying change in net asset value, the change in the index and the idiosyncratic shock.

From equations (23) and (40), and incorporating the payout on the mutual fund assuming that it is reinvested[14], the total return on M_t is given by

$$\begin{aligned} R^M_{t,t+\tau} &= e^{[c(1+\theta)-\nu-\theta\Sigma)]\tau} \left(\frac{M_{t+\tau}}{M_t}\right), \\ &= \tilde{K}_{t,t+\tau} \left(\frac{{R^S_{t,t+\tau}}^{1+\theta}}{{R^I_{t,t+\tau}}^{\gamma\theta}}\right), \end{aligned} \tag{41}$$

where $\tilde{K}_{t,t+\tau} \equiv e^{\left[(1+\theta)(c-\frac{1}{2}\theta\Sigma)-\frac{1}{2}\theta\gamma(\theta\gamma+1)\sigma_I^2+\theta r(\gamma-1)-\frac{1}{2}\sigma_N^2\right]\tau+\sigma_N\left(\tilde{Z}_{N,t+\tau}-\tilde{Z}_{N,t}\right)}$.

[14] From equation (23), this is paid at an annualized fractional rate

$$cV_t/M_t = c(1+\theta) - \nu - \theta\Sigma.$$

From a risk point of view, the sensitivity of the total return on M_t to the fund's NAV return can be written as:

$$\frac{\partial R^M_{t,t+\tau}}{\partial R^S_{t,t+\tau}} = (1+\theta)\tilde{K}_{t,t+\tau}\left(\frac{{R^S_{t,t+\tau}}^{\theta}}{{R^I_{t,t+\tau}}^{\gamma\theta}}\right). \tag{42}$$

The risk of the fund is not constant as long as the net flows are sensitive to fund performance, i.e., θ is nonzero. Thus, returns on the fund's revenue stream are nonlinear in the fund's NAV return relative to the benchmark.

To gauge the magnitude, it is convenient to factor out the idiosyncratic fund flow noise. Specifically, consider $E_t\left[R^M_{t,t+\tau} \mid R^S_{t,t+\tau}, R^I_{t,t+\tau}\right]$. Figure 1A graphs the return on the fund's value as a function of the return on the NAV of the fund and the excess performance of the fund for different values of θ.[15] The graph makes clear a few salient points. First, as θ increases, the return on the fund's value increases as a function of the excess return on the benchmark, $\frac{R_s}{R_I}$. Second, this increase occurs at an increasing rate. Third, in order to see the convexity more clearly, Figure 1B graphs cut-throughs of the relation between the return on the fund's revenue stream and R_S (holding R_I constant). At $\theta = 0$, the relation is linear by construction as net flows are insensitive to fund performance. As θ increases, however, the convexity is very transparent for large moves in the NAV return. This result implies that if a mutual fund business needs to hedge its risk, it must do so either dynamically or using volatility-based instruments (like options) to capture the convexity. The intuition is very similar to delta and gamma hedging with respect to options and the underlying stock.

3 Fund Flows

The theoretical model in Section 2 captures the important stylized fact that net flows into the fund depend on the underlying fund's performance. This stylized fact induces a convex relation between the mutual fund's value and its net asset value. While this results captures the most important component of the valuation, the specification was fairly simple in order to generate closed-form solutions.

Because there has been a considerable effort in the literature to document the relation between fund flows and performance, it is worthwhile incorporating these results into our framework.[16] Most important, this literature identifies a number of characteristics that ren-

[15] We assume that $\tau = 1$, $\sigma = 0.20$, $\sigma_I = .15$, $\rho = 0.75$, $\gamma = 1$ and $\nu = 0$. These parameters are chosen to coincide roughly with those governing the equity asset class, evidence of which is presented in the next section.

[16] See, for example, Patel, Zeckhauser, and Hendricks (1991), Ippolito (1992), Warther (1995), Gruber (1996), Chevalier and Ellison (1997), Remolana, Kleiman and Gruenstein (1997), Sirri and Tufano (1998),

der closed-from solutions intractable. Fortunately, we can perform a Monte Carlo simulation of the possible paths of the fund's NAV and keep track of important features such as the fund's size and its most recent performance. We can build quite complex, and empirically realistic, path-dependencies and nonlinearities into the relation between flows and performance. In the remainder of this section we develop a model of empirical fund flows and estimate the model on the universe of equity mutual funds over the period 1970 to 2002.

3.1 Data

The data employed in this study covers annual data on mutual funds over the period 1970-2002. The source of the data is CRSP's survivorship-free database on US mutual funds. The advantage of this database is that it includes funds that have disappeared during the sample period, such as through mergers or liquidations. It was originally compiled by Mark Carhart and is described in detail in his studies on mutual fund performance (Carhart (1995,1997)). The file includes a number of items of interest for each mutual fund. In particular, the fund's return, total assets under management, fee structure, age, and description are provided on a quarterly basis. Because we are only interested in active management, we use the breakdown in Elton, Gruber and Blake (2004) to remove all index funds from the sample.

As has been pointed out by others, the database is not free of errors. For example, Elton, Gruber and Blake (2001) argue that, though in principle the data is survivorship-free, missing observations lead to some similar problems. Moreover, they find other errors, including those associated with mergers and splits. This is potentially important as mergers and splits will affect the measure of net flows into the funds. As a result, we employ several filters to address these concerns.

First, we only consider funds with a beginning of year assets under management of $10 million or greater. Second, we filter out observations with either extreme net flows or extreme returns. We filter out observations in the 5% tails of the net flows distribution and the 1% tails of the return distribution. A close examination of a subset of these filtered observations indicates that many are either recording errors or errors resulting from mishandling of fund mergers and splits. Third, we eliminate observations for which the data on the year in which the fund was organized is clearly incorrect, e.g., data for the fund exists in years prior to the recorded fund organization year. Finally, we eliminate observations that have missing data for one or more of the variables–flows, returns, age, size, and fees.

Table 1 provides a summary of the main characteristics of mutual funds studied in this paper. Specifically, Table 1 looks at the distribution of returns, flows, size, fees, and age. We

Edelen (1999), Bergstresser and Poterba (2002), and Del Guercio and Tkac (2002), among others.

also report the tracking error volatility against each fund's particular benchmark. (Note that we define the benchmark as the median return of a fund's particular asset class.) Several observations are in order. First, the key variables for determining the net flow dynamics are the age, size and fee of the fund, in this case, median values of 7 years, 113.4 million dollars and 1.16%, respectively. To the extent that fund age and size tend to diminish both the net flow growth rate and sensitivity to fund performance (e.g., Chevalier and Ellison (1997)), the distribution of a fund's age and size across the sample will be an important determinant of value. Second, the important parameter governing mutual funds is the relation between the NAV returns and the corresponding benchmark returns, i.e., the volatility of the tracking error. For this sample, the average NAV and benchmark return volatility are 19.6% and 14.5%, respectively with an average correlation of 0.762. This leads to a tracking error volatility of 12%.

3.2 Specification for Fund Flows

The first step is to choose a tractable model for the empirical relation between net flows and fund performance. We measure fund performance relative to a benchmark defined as the median return from the asset class (e.g., Sirri and Tufano (1998)). Consistent with the literature, we look at the recent performance of the fund over the past year.[17] The reason for looking at lagged one-year returns is the belief in the literature that investor behavior is somewhat sticky as it applies to investing in or removing monies from mutual funds.

We consider the following econometric specification[18]:

$$\begin{aligned} log\left(\frac{N_{it+1}}{N_{it}}\right) &= [\nu(X_{it})] + F\left[\left(\frac{\Delta S_{i;t-1,t}}{S_{it-1}} - \frac{\Delta I_{t-1,t}}{I_{t-1}}\right), \theta(X_{it})\right] + \epsilon_{it} \\ \epsilon_{it}^2 &= \upsilon(X_{it}) + \eta_{it}, \end{aligned} \quad (43)$$

where i subscripts the fund, X_{it} is a vector of the fund's characteristics at time t, N is the fund's flow, ΔS is the change in the fund's NAV, and ΔI is the change in the index's

[17] We also analyze the current performance and long-run performance of the fund to better understand the fund-flow dynamics. We find that both the recent performance and long-term performance contribute to the regression's explanatory power. The effects, however, are second order to the recent lagged return. Moreover, to the extent fund performance is not highly autocorrelated, these additional effects do not have much impact on that of the one-year lagged return. Since these effects only serve to increase the sensitivity of net flows to fund performance, we consider only the lagged return results to coincide with previous analyses.

[18] We ignore evidence of macro factors affecting fund flows (e.g., Warther (1995), Chevalier and Ellison (1997) and Sirri and Tufano (1998)). While the choice of variables differs across these studies, there is a general idea that a flow variable either for the fund's asset class or the market in general has explanatory power. If this variable is not correlated with the characteristics of the fund, or that fund's excess performance, however, then our estimates will be unbiased.

value. For the analysis to follow, we assume $\nu(\cdot)$ and $\theta(\cdot)$ are quadratic functions of the fund characteristics, and $F[\cdot]$ is piecewise linear (and thus nonlinear) in excess performance.[19] The second equation takes into account the fact that there is heteroskedasticity across the funds that depends on the characteristics. For example, the variance of the random flow is probably less for large, old funds than for emerging, small ones.

With respect to the fund's characteristics, we consider the following variables for X_{it}: (i) log size of the fund, (ii) age of the fund, and (iii) annual percentage fees. (For previous analyses using these variables, see Chevalier and Ellison (1997), Sirri and Tufano (1998), Del Guercio and Tkac (2002) and Bergstrasser and Poterba (2002), among others). In brief, these authors tend to find that (i) ν declines with size and θ is invariant, (ii) ν and θ both decline with age, and (iii) fees decrease ν but increase θ. Our specification is somewhat more general as it allows for numerous nonlinear interaction effects between fund flows, fund performance and fund characteristics. As we shall see from a statistical view point in Section 3.3, and then economically in Section 4, even subtle differences in the specification can play an important role.

Note that we do not incorporate investor rationality into our framework beyond what is implied by the empirical model for net flows. An analogous debate can be found in the mortgage-backed security literature with respect to empirical prepayment and rational prepayment models. Though we discuss this issue briefly in the conclusion, two areas seem potentially important. The first is the tax overhang issue looked at by Barclay, Pearson and Weisbach (1998) and Bergstresser and Poterba (2002). They show that the flow of monies into and out of a fund are affected by its tax overhang. To the extent that tax overhang might be related to the fund's age or size, this could impact our coefficient estimates. That stated, Bergstresser and Poterba (2002) find that the size and age variables are statistically important regardless of the tax burden. The second is the market's "assessment" of a fund's alpha. Recent theoretical work by Berk and Green (2002) provides a rationale for fund flows in a world where managers are skilled. Empirically, there is some evidence to suggest that a fund's Morningstar rating, or similar measure of the fund's ex ante alpha, may be important (e.g., Del Guercio and Tkac (2002) and Bergstresser and Poterba (2002)). This paper abstracts from the debate about the alpha of the fund and how it impacts pricing.

3.3 Empirical Results

Table 2 provide the regression results for net flows. For both net flows and the variance of the random shocks to these flows, we provide the parameter estimates, standard errors,

[19] We assume that the quadratic function truncates at its minimum/maximum value, so that the function is monotonic and nonlinear in the characteristic.

number of fund observations and R^2s. The estimation is performed for three models, $\theta = 0$, θ equals a constant and θ given by equation (43).[20]

As an illustration of how to understand the results, first consider the growth rate estimates from the net flow regression. These estimates, $\hat{\nu}$, are functions of a constant and the three characteristics size, age and fees. For the full-blown regression, growth rates are decreasing in fees, size and age. Consider the life cycle of a typical fund with median fee of 1.16% that goes from 25th percentile in age and size to 75th percentile in these characteristics (i.e., from 4 to 16 years and $40 million to $384 million, respectively). Its growth rate will decline from 8.48% to -9.38%. Recalling the closed-form solutions for valuation in Section 2, these effects clearly have a profound impact on valuation.

Table 2 shows evidence for a nonlinear relation between fund flow and performance. Consider the regression results in which θ does not depend on fund characteristics. The coefficient on negative performance is 0.66 versus 0.90 for positive performance, which highlights both the positive correlation and the convexity of the fund-performance relation. It is economically quite important as the R-squared jumps from 13.85% to 24.22% when we add performance as an explanatory variable for net flows. Similar to the exercise above for growth rates, we can analyze the effect of the characteristics on the flow-performance sensitivity parameter, θ. In moving from the 25th percentile in age and size to the 75th percentile, for positive performance the sensitivity drops from 1.38 to 1.089 whereas for negative performance it drops from 0.66 to 0.52. The convexity is still present yet the magnitudes have dropped 20% or so. Thus, through the life cycle of a fund, the performance-flow relation plays less of a role.

In general, irrespective of the direction of the fund's performance, this sensitivity declines in age and increases in fees. The effect of age can be described as either a "burnout" effect similar to that of mortgage-backed securities or agents learning about the fund manager's ability and thus being less sensitive to idiosyncratic performance. The effect of fees is more difficult to discern. Sirri and Tufano (1998) argue that higher fees tend to be associated more with funds who market their funds; alternatively, in investor's minds, high fees might be a red flag to monitor performance. The most interesting effect, however, is that of size. For negative performance, the sensitivity increases in size, while for positive performance it decreases. For example, as a fund grows old, say from the 25th to the 75th percentile, the sensitivity jumps 0.37 versus dropping -0.30 for negative and positive performance, respectively. This change is enough to make the functional form change from convex to concave,

[20]Note that the standard errors reported are derived under standard OLS assumptions. These standard errors are most probably understated due to the correlation across contemporaneous fund flow residuals arising from macro factors, amongst other variables. To the extent that this paper uses only the point estimates in deriving the mutual fund's valuation and risk properties, this issue is peripheral to our analysis.

which can potentially have important effects for fund managerial incentives, optimal choice of fees, and valuing the funds' cash flows.

Table 2 also reports results for the regression of the squared residuals from the net flow regression on a constant and characteristic variables size, fees and age. The key feature of the results is that the variance of idiosyncratic flow shocks is decreasing in both fund size and age. Consider our example of moving from the lower quartile to the upper quartile of funds. The volatility of annual flow shocks drops from 8.6% to essentially 0%.[21] In other words, net flow shocks may induce large amounts of volatility in the value of small and/or young funds, but as these funds age and grow, flows become much more predictable and the volatility will be determined principally by return shocks.

4 Applications

4.1 Numerical Valuation Methodology

Valuing the stream of cash flows to the fund manager is equivalent to calculating the expectation in equation (17), which can be written as

$$M_t = \int_t^\infty E_t\left[e^{-r(\tau-t)} c V_\tau\right] \, d\tau.$$

Intuitively, this says that the value of the asset equals the sum of expected values of discounted cash flows that it produces.[22] The dynamics of V_t are driven by the dynamics of S_t and I_t (given in equations (2), (3) and (4)), and by the performance related fund flows given in equation (7). We can estimate this expectation numerically by simulating (discrete) paths for S_t and I_t, calculating the fee paid and the change in assets each period, discounting the fees back along each path, and finally averaging over a large number of paths. This is the basis of all Monte Carlo valuation techniques (see, for example, Boyle (1977) or Jacob, Lord and Tilley (1987)), and allows us to value the fund taking into account an arbitrary dependence of cash in- and outflows on the history of asset returns.

To implement the valuation procedure, we simulate each path using a discretized version

[21] For any given set of values, the fitted value maybe negative and so is truncated at zero.

[22] Note that all expectations are taken relative to the risk-neutral probability distribution.

of equations (1), (1) and (4)),

$$S_{t+(i+1)\,\Delta t} = S_{t+i\,\Delta t} \exp\left[\left(r - c - \frac{1}{2}\sigma^2\right)\,\Delta t + \sigma\sqrt{\Delta t}\,\tilde{\epsilon}_i^S\right], \quad (44)$$

$$I_{t+(i+1)\,\Delta t} = I_{t+i\,\Delta t} \exp\left[\left(r - \frac{1}{2}\sigma_I^2\right)\,\Delta t + \sigma_I\sqrt{\Delta t}\,\tilde{\epsilon}_i^I\right], \quad (45)$$

where $\tilde{\epsilon}_i^S$ and $\tilde{\epsilon}_i^I$ are normal random variables with mean 0, variance 1, and correlation ρ.[23] Given this path, we use equation (43) to determine the value of N_t at each time step, then calculate the cash flow each period as a multiple of the current assets under management, $N_t S_t$. We repeat this many times, and average across paths.

To improve the accuracy of our estimate of M_t, we use the antithetic variate approach. Instead of drawing each path independently, we draw paths in pairs. The first path in each pair is determined by a sequence of i.i.d. standard normal random numbers $\tilde{\epsilon}_1, \tilde{\epsilon}_2, \ldots$. The second path is generated using the random numbers $-\tilde{\epsilon}_1, -\tilde{\epsilon}_2, \ldots$. The random numbers that generate each path are from the correct distribution,[24] but this procedure generates negative correlation between the present values obtained from each path, lowering the standard error of the estimate of V_t (see Boyle (1977)).

4.2 Results

Tables 3A and 3B report the present value of the fund's fees as a fraction of current assets under management, i.e., $\frac{M_t}{V_t}$, the length of time it takes to reach half the present value of the revenue stream, and the distribution of the realized present values across 100,000 simulated paths. These calculations are performed for the fund with median characteristics as well small and young, and, conversely, large and old.

As a benchmark comparison, Table 3A looks at the valuation and distribution of $\frac{M_t}{V_t}$ assuming that the fund flow sensitivity parameters, θ, are either zero, constant (i.e., θ does not depend on the characteristics) or follow the empirical specification in Section 3.[25] A quick glance at Table 3A might suggest that the sensitivity parameter is not important. This

[23] To generate these, we generate two independent standard normal random variables, $\tilde{e}_i^S$ and $\tilde{e}_i^I$, and then calculate

$$\tilde{\epsilon}_i^S = \tilde{e}_i^S,$$
$$\tilde{\epsilon}_i^I = \rho\tilde{e}_i^S + \sqrt{1-\rho^2}\tilde{e}_i^I.$$

[24] If $\tilde{\epsilon}$ is $N(0,1)$, so is $-\tilde{\epsilon}$.

[25] Note that for each of the two benchmark cases, the system is re-estimated to maintain the unconditional distribution of net flows.

conclusion, however, would be misleading. Recall that the parameter θ has two offsetting effects in the net flow dynamics. The first effect arises from NAVs falling over time due to fees; thus, the sensitivity drives down flows and $\frac{M_t}{V_t}$s. The second effect leads to a convex payoff between fund values and fund NAVs, leading to higher $\frac{M_t}{V_t}$s. For the parameter values estimated here, these effects are similar in magnitude.

One of the more striking aspects of Table 3A are the magnitudes of $\frac{M_t}{V_t}$. With empirically relevant data, the values range from 16% to 105% to 35% of the assets under management for the old, young and median funds, respectively. While these values are net of costs (e.g., employees, marketing, leases, etc..), they still represent large transfers from investors to the various constituencies of the fund. One argument for why these transfers might be smaller in the future is that the growth rates documented in Table 2 cannot be sustained going forward. In other words, the mutual fund business has developed into a fully mature sector, and future overall growth in the industry will be more closely tied to future growth in the aggregate economy. We believe this is an interesting area for future research.

Table 3B extends this analysis by incorporating other parameter values and characteristics. We look at parameter values at the 25th and the 75th percentile as representative of small and large parameter values. Fund values are rising in fees and tracking error variance, and falling in size and age. For example, suppose the median fund were to implement a tracking error volatility of 25% versus 5% (i.e., 75th versus 25th percentile of distribution). This would lead to an increase in $\frac{M_t}{V_t}$ from 0.33 to 0.42. Interestingly, although the present values differ, their median $\frac{M_t}{V_t}$ values are similar, e.g., 0.26 versus 0.27. This suggests that not only is the distribution of mutual fund values highly skewed, but that this skewness is the source of the difference in value across funds. In order to understand why this occurs, note that at the tail of the distribution, extraordinary high excess return performance will lead to a quick build-up of assets under management, which explains the 95% tails of 0.77 versus 1.18. This skewness is associated though with over twice as much volatility in the fund's value, e.g., 0.55 versus 0.25. [26] This is important because to the extent fund managers care about their distribution of $\frac{M_t}{V_t}$s, the higher $\frac{M_t}{V_t}$ comes at a cost not priced by the market. In theory, the manager's compensation contract with the mutual fund company should possibly take this feature into account.

Note that the specification and estimation of fund flows in equation (42) and valuation results in Table 3B point to the importance of both the size and age of the fund. To understand the impact more fully, Figure 2 graphs $\frac{M_t}{V_t}$ against size and age. *STILL TO BE COMPLETED.*

[26] Note that the higher variance in flows leads to more fund deaths. Unlike our closed-form solution, we treat N_t, the number of shares, as an absorbing barrier at zero. That is, any fund hitting zero shares dies.

4.2.1 Optimal Volatility of Tracking Error Theory and Evidence

Suppose fund managers were to choose the volatility of tracking error. What would their optimal policy be in the context of the valuation model? Under the theoretical model of Section 2 and analysis given in 2.1, they have an incentive to increase this volatility by as much as possible. However, under the empirical specification estimated in Section 3, this is no longer necessarily the case. In fact, the previous results showed that large funds lose flows at a greater rate than gain flows when producing negative versus positive performance. Thus, greater tracking error volatility might actually hurt the fund's growth prospects and thus present value of future fees. Of course, as illustrated by Table 2, the relation is a complicated function of age, and age and size are not independent.

This intuition is confirmed by Figure 3 which graphs the value of $\frac{M_t}{V_t}$ as a function of Σ for small, young versus large, old funds. When a fund is small, it pays to take large bets because (i) the flow-performance relation tends to be convex in performance, and (ii) the convexity effect is most relevant when it can be combined with funds that have natural growth in order to create the "cash cow". The exact opposite exists for large funds because there is little natural growth left and, as noted above, the flow-performance relation is actually concave in size (and enough so to make a difference).

TO BE COMPLETED - solve via simulation the optimal volatility path using a discrete number of volatility choices.

TO BE COMPLETED - test whether in fact the volatility of tracking error declines with size and correspondingly test for its independent effect on age. Preliminary evidence supports the theory. Some may argue this may have something to do with the scalability of active versus passive investing, but this papr provides an alternative rationale.

4.2.2 Optimal Fee Policy Theory and Evidence

The theory of Section 2 and, in particular, 2.2, showed that high fees produce higher short-term revenues against (i) lower NAVs due to fees, (ii) lower estimated growth rates in flows due to higher fees, and (iii) lower growth rates due to the flow-performance sensitivity's implicit effect of fees driving down NAVs. The same analysis can be applied to the simulation of the empirically estimated fund flow specification. Table 3B shows that, for our data, $\frac{M_t}{V_t}$ increases over some range in fees for the median fund. This is not particularly surprising because, for $\frac{M_t}{V_t} < 1$, you would want to extract all the assets immediately. Of course, this is not feasible in practice. Recall the differences of the fee effect between the infinite horizon and finite horizon models of Section 2; the results here serve to show that the changes in parameter values are paramount to shortening the life of the fund.

TO BE COMPLETED - solve via simulation the optimal fee path using a discrete number of fee choices.

TO BE COMPLETED - test whether in fact the fee declines with age and test for its independent effect on size. Preliminary evidence supports the theory. However, it is complicated by the fact that fund costs (leases, marketing, some employees, etc..) are most likely much higher for small than large funds due to their fixed nature.

4.2.3 Fundamental Determinants of Mutual Fund Companies Theory and Evidence

The valuation framework in Sections 2 and 4 provide predictions on the relative value of particular funds (or more generally, fund families) and on how these values change through time as a function of the important variables. We have identified three sources of variation: the return on NAV, the excess return on NAV relative to a benchmark, and random shocks to net flows. Moreover, these variables enter nonlinearly and in a state-dependent way as a function of the fund's current characteristics. An interesting examination would be to look at the sales of mutual fund companies or the behavior of publicly traded fund companies to see how well the model captures these basic elements cross-sectionally. For example, do we find that most of the return variation can be explained by fund performance and the underlying asset movements?

In order to understand the return behavior of mutual fund values using the empirically relevant model in Section 3, we calculate for each of 10,000 paths the next period's present value of the revenue streams for each fund. This next period's value requires us to replicate the original procedure along each path, that is, to estimate the expected value using 10,000 possible paths starting from each of the 10,000 realizations next period, for a total of 10 million paths. We then calculate the return on the fund's value, the NAV and the benchmark and estimate the relation between these variables using our 10,000 return values.

Table 4 reports results from the regression of the fund value returns (R^M) on contemporaneous returns on the NAV (R^S) and the benchmark (R^I), including nonlinear terms. The regressions use 10,000 simulated returns for the median characteristics with the exception of age, which takes on three different values–zero years, seven years (the median age), and fourteen years. For each regression, we report the coefficients (with corresponding standard errors below) and the R^2. The results illustrate three important features of the simulated data.

- asset returns exhibit nonlinearity that decreases with age.
- asset returns exhibit a substantial amount of systematic risk, which can be seen from

regressions against only benchmark returns.

- the risk, however, comes primarily from the component due to NAV returns that are uncorrelated with the benchmark and shocks to net flows.

Our model has strong predictions about the value of asset management firms. In order to examine those implications we collected data on stock returns of publicly traded asset managers. In particular, we start our sample selection with a universe of three relevant four-digit SIC codes, and corresponding NAICS. We narrow an initial sample of 63 firms down to a sample of 27 firms by restricting ourselves to companies that are primarily in the mutual fund management business. Multiple sources, including companies' website, Bloomberg, Yahoo-Finance and Lexis-Nexis were used to make this judgment. For each company in this sample we identify within the CRSP Mutual Fund Database all the mutual funds that were managed by it. For each such mutual fund we extract fund information such as monthly return and assets under management for the relevant period. We include equity and fixed income funds, and exclude the rest. We identify such funds by their policy codes. Monthly returns for the asset management companies are obtained from the CRSP monthly stock database. The sample period is January 1992 to December 2002, for a maximum sample size of 132 months. We run our regressions fund-by-fund as well as on a portfolio ("industry) level. In addition to our full sample of 27 firms, we also look at a narrow sample of nine firms with at least 100 monthly observations. This significant decline in sample size is due primarily to the fact that many companies in our sample disappeared due to a wave of mergers in the industry during the 90's.

We run the following three regressions:

$$R^i_t = \alpha^i + \beta^i_1 R^{sp,t} + \beta^i_2 R^{leh,t} + \epsilon_t \tag{46}$$

$$R^i_t = \alpha^i + \beta^i_1 \Sigma_j w^j_t R^j_t + \beta^i_2 \Sigma_j w^j_t (R^j_t R^j_{bench,t}) + \epsilon_t \tag{47}$$

$$R^i_t = \alpha^i + \beta^i_1 \Sigma_j w^j_t R^j_t + \beta^i_2 \Sigma_j w^j_t (R^j_t R^j_{bench,t}) + \beta^i_3 \Sigma_j w^j_t F^i_t + \epsilon_t \tag{48}$$

where $i = 1, 2, , 27$. We also run the regressions on the aggregate portfolio (27 firms) level and the narrow (9 firms) portfolio level. The first regression is a standard two factor market model with the S&P500 and the Lehman aggregate indexes. This regression provides us with a baseline set of regression results. In the second regression we specify the return generating process for the fund management firm as a function of fundamentals, motivated by our theoretical model. Specifically, returns are a function of the weighted return of the funds that the firm manages, where the weights are related to AUM.[27] In particular,

[27] The weights should in theory be a function of the age, size and fees of the fund. We intend to incorporate these factors in the next version.

$w^j = AUM_t^j/AUM_t$, where AUM_t is fund i's total assets under management (we omit the i index for ease of notation). Returns are also a function of the aggregate excess performance of the firm's funds, where excess performance is relative to a benchmark. As noted above we narrow our set of funds to equity and fixed income funds. The relevant benchmarks are the S&P500 index and the Lehman aggregate respectively. The third regression takes the natural next step in the context of our model and adds in the effect of flows. Flows are aggregated across all funds within a firm, and calculated as log AUM growth less log returns, $F_t^i = \Sigma_j[log(AUM_t^j) - log(AUM_{t-1}^j R_t^j)]$.

In Table 5, we report the results for these three regressions. We use quarterly frequency in order to minimize the effects of nontrading, the timing of AUM reporting and other possible data problems. In Panel A we provide the baseline regression results. Market betas are close to and below unity for the full sample and for the equally weighted portfolio of funds in that sample, and for the narrower sample of 9 firms with a relatively larger and reliable sample size the beta is generally lower, with a median beta of 0.66 and a portfolio beta of 0.82. The median baseline R^2 is 12% for the narrow sample, and 41% for the portfolio.

The second regression β_1 coefficients are above one, as predicted by theory, with a median value for the narrow sample of 1.7, and 1.36 for the narrow portfolio. The median R^2 is higher than for the baseline regression, namely, 15.5%, while comparable for the portfolio. The fact that the improvement in explanatory power is exhibited on the firm level, but not on the aggregate portfolio level, is intuitive and compelling for our story we capture the effect of fundamental, be it market-related or systematic. This adds value relative to the baseline regressions at the firm level, but not on the aggregate portfolio level. Similar and stronger results are observed in the third regression, when flows are brought in. At the firm level the median R^2 for the narrow sample is now up to 23.2%, nearly twice that of the baseline market model regression.

In sum, the results are strong and consistent with our model. It is interesting to note that the increase of explanatory power from the baseline median of 12% and up to 23.2% is reached with relatively simplistic models of fundamentals' effects on value. There are many possible improvements to this model which are left for further research. It is useful to note, though, how even a simple attempt to capture the effect of fundamental leads to an immediate increase in explanatory power. This kind of results is relevant and instructive in the context of the market efficiency literature.

5 Concluding remarks

We develop a valuation methodology for the fee flow generated by asset managers using a contingent claims approach. A closed form formula shows the dependence of fund value not only on the usual suspects such as fees, current asset value and expected asset growth, but also on asset return volatility and the flow- performance sensitivity. Motivated by economic intuition developed from theory we estimate typical fund parameters for fund flows. Using Monte Carlo simulation, we can go outside the confines of the closed form formula and value funds where such critical parameters as growth and flow-performance sensitivity are age and size dependent. We show the critical importance of fund characteristics for fund valuation in an economically meaningful environment.

We then apply this framework to several key issues in asset management, including the risk-taking incentives of mutual funds, the value of active investing, the optimal contract in terms of fees, and the fundamental determinants of mutual fund valuation. Within these applications, some new insights are developed. As examples, we derive the optimal fee and tracking error volatility paths for mutual fund managers. Using data on mutual funds, we show that the data are broadly consistent with the implications.

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Table 1: Descriptive Statistics

A: Fund Characteristics

	Mean	SD	Percentile 5	25	50	75	95
Fees (%)	1.25	0.54	0.48	0.88	1.16	1.59	2.20
Age (yrs)	13.39	14.51	2.00	4.00	7.00	16.00	48.00
Size ($M)	641.51	2779.40	14.00	40.07	113.14	384.76	2312.96
Returns (%)	7.67	19.43	-25.42	-6.07	9.35	21.63	36.54
Flows (%)	12.79	46.47	-24.99	-11.81	-1.36	19.16	99.09

B: Benchmark Returns

SD	Corr.
0.145	0.762

Panel A presents means, standard deviations and percentiles for the fees, age, size, returns and flows for the funds. Panel B presents the standard deviation of the benchmark return and the correlation of this return with the NAV returns.

Panel A: Fund Flow Regression

		Zero		Constant		Full	
	Variable	Coeff.	(SE)	Coeff.	(SE)	Coeff.	(SE)
Growth	Const	0.6423		0.6275		0.5999	
	Size	-0.0765	(0.0061)	-0.0715	(0.0057)	-0.0593	(0.0082)
	Fee	-0.0358	(0.0102)	-0.0405	(0.0096)	-0.0542	(0.0136)
	Age	-0.2748	(0.0092)	-0.2701	(0.0087)	-0.2835	(0.0123)
	Size^2	0.0051	(0.0006)	0.0052	(0.0005)	0.0048	(0.0008)
	Fee^2	-0.0007	(0.0033)	0.0021	(0.0031)	0.0096	(0.0044)
	Age^2	0.0401	(0.0019)	0.0402	(0.0018)	0.0423	(0.0026)
θ^-	Const			0.6611	(0.0281)	0.4363	(0.2927)
	Size					0.1780	(0.0939)
	Fee					0.4253	(0.1583)
	Age					-0.6853	(0.1430)
	Size^2					-0.0071	(0.0087)
	Fee^2					-0.0506	(0.0461)
	Age^2					0.0981	(0.0311)
θ^+	Const			0.8996	(0.0224)	1.2489	(0.2399)
	Size					-0.1504	(0.0716)
	Fee					0.7193	(0.1459)
	Age					-0.2191	(0.1142)
	Size^2					0.0074	(0.0064)
	Fee^2					-0.2131	(0.0448)
	Age^2					0.0208	(0.0256)
	$R2$	0.1385		0.2422		0.2504	
	Obs.	25,860		25,860		25,860	

Panel B: Volatility of Fund Flow Regression

	Zero		Constant		Full	
Variable	Coeff.	(SE)	Coeff.	(SE)	Coeff.	(SE)
Const	0.2354	(0.0123)	0.2320	(0.0111)	0.2300	(0.0110)
Size	-0.0169	(0.0039)	-0.0180	(0.0035)	-0.0181	(0.0035)
Fee	0.0242	(0.0064)	0.0097	(0.0058)	0.0106	(0.0057)
Age	-0.0793	(0.0058)	-0.0716	(0.0052)	-0.0695	(0.0052)
Size^2	0.0008	(0.0004)	0.0009	(0.0003)	0.0009	(0.0003)
Fee^2	-0.0051	(0.0021)	-0.0030	(0.0019)	-0.0038	(0.0019)
Age^2	0.0099	(0.0012)	0.0091	(0.0011)	0.0087	(0.0011)
$R2$	0.0489		0.0463		0.0457	
Obs.	25,860		25,680		25,680	

Table 2: Fund Flow-Performance Regression

Results from the estimation of equation (43). Panel A presents results from the piecewise linear regression of annual net flows on lagged annual returns where the constant and regression coefficient depend nonlinearly on log size, fee and log age. Panel B presents results from the regression of the squared fitted residual from Panel A on the same characteristics.

Table 3: Valuation Summary Statistics

A: Value vs. regression specification

	Type 1			Type 2			Type 3		
	Default	$\theta =$ const.	$\theta = 0$	Default	$\theta =$ const.	$\theta = 0$	Default	$\theta =$ const.	$\theta = 0$
M/V	0.365	0.408	0.373	1.1	1.21	1.11	0.166	0.204	0.189
Vol	0.311	0.678	0.31	1.13	2.29	1.13	0.0983	0.276	0.12
Skew	4.58	40.4	4.81	5.84	43	6.18	2.83	28.3	3.58
Half	16	17.3	16.1	20.8	22.3	20.6	20.6	12.2	11.2
5%	0.111	0.0941	0.117	0.269	0.223	0.272	0.0669	0.0568	0.0762
25%	0.186	0.166	0.194	0.494	0.434	0.501	0.102	0.0931	0.115
50%	0.279	0.264	0.289	0.786	0.731	0.797	0.141	0.141	0.158
75%	0.436	0.449	0.446	1.3	1.31	1.31	0.2	0.228	0.224
95%	0.9	1.13	0.899	2.94	3.51	2.92	0.346	0.536	0.4
99%	1.55	2.46	1.56	5.47	8.05	5.45	0.523	1.09	0.639

B: Value vs. parameter estimates

	$\sigma = 0.15055$	$\sigma = 0.2875$	Fee = 0.0088	Fee = 0.0159	Age = 4	Age = 16	Size = 40.07	Size = 384.76
M/V	0.331	0.419	0.291	0.465	0.453	0.29	0.535	0.275
Vol	0.248	0.547	0.252	0.385	0.432	0.211	0.456	0.229
Skew	3.63	10.8	4.71	4.4	5.13	3.91	4.72	4.29
Half	15.3	16.5	16.5	15.3	16.1	15.7	19.3	13.2
5%	0.111	0.0955	0.0871	0.146	0.122	0.102	0.161	0.0838
25%	0.181	0.172	0.147	0.242	0.213	0.163	0.274	0.14
50%	0.264	0.275	0.221	0.359	0.331	0.234	0.411	0.21
75%	0.4	0.473	0.346	0.557	0.538	0.348	0.641	0.33
95%	0.774	1.18	0.724	1.13	1.18	0.664	1.31	0.674

Table 3A and 3B provide summary statistics for Monte Carlo simulations of Mutual Fund Valuation. For each fund Type (median – 1, young/small – 2 and old/large – 3), 100,000 Monte Carlo simulations were performed. The table shows the resulting estimate of M/V, the half-life of the fund (defined as the time to maturity of a fund whose value equals 50% of the value of a fund with a life of 100 years), and the 5%, 25%, 50%, 75% and 95% quantiles of the total present values of cash flows for each path. Panel A. looks at three different specifications of the fund-flow regression, while in panel B. the full-blown estimation is used for different sets of parameter values.

Table 4: Simulated Return Regression Results

Const	R^S	${R^S}^2$	R^I	${R^I}^2$	$R^S R^I$	R^2
			Age=0 years			
0.0444	1.3697					0.2780
0.0032	0.0156					
0.0299	1.2878	0.4313				0.2803
0.0036	0.0186	0.0535				
0.0505			0.9218			0.0755
0.0037			0.0228			
0.0502			0.9187	0.0181		0.0755
0.0041			0.0286	0.1008		
0.0658	1.9005	0.5341	-0.9676	-0.5438		0.3223
0.0037	0.0273	0.0652	0.0364	0.1081		
0.0615	1.9196	0.7781	-1.0056		-0.7365	0.3223
0.0037	0.0264	0.1029	0.0329		0.1480	
0.0639	1.9070	0.6728	-0.9793	-0.3104	-0.3728	0.3223
0.0040	0.0277	0.1250	0.0375	0.2095	0.2866	
			Age=7 years			
0.0234	1.0393					0.3311
0.0021	0.0104					
0.0167	1.0015	0.1991				0.3322
0.0024	0.0125	0.0359				
0.0229			0.7974			0.1169
0.0025			0.0155			
0.0227			0.7953	0.0124		0.1169
0.0028			0.0194	0.0685		
0.0357	1.3340	0.2366	-0.5282	-0.2433		0.3570
0.0025	0.0185	0.0442	0.0246	0.0732		
0.0338	1.3432	0.3322	-0.5471		-0.3069	0.3570
0.0025	0.0179	0.0697	0.0223		0.1002	
0.0353	1.3354	0.2669	-0.5307	-0.1923	-0.0815	0.3570
0.0027	0.0188	0.0847	0.0254	0.1419	0.1941	
			Age=14 years			
0.0333	1.0080					0.3335
0.0020	0.0101					
0.0273	0.9741	0.1790				0.3343
0.0023	0.0120	0.0346				
0.0328			0.7748			0.1181
0.0024			0.0150			
0.0328			0.7742	0.0032		0.1181
0.0027			0.0188	0.0661		
0.0456	1.2953	0.2127	-0.5107	-0.2281		0.3591
0.0024	0.0178	0.0426	0.0238	0.0707		
0.0439	1.3040	0.3006	-0.5287		-0.2848	0.3590
0.0024	0.0172	0.0672	0.0215		0.0967	
0.0453	1.2965	0.2372	-0.5128	-0.1869	-0.0658	0.3591
0.0026	0.0181	0.0817	0.0245	0.1369	0.1873	

The table presents results from regressions of fund value returns on returns, squared returns and an interaction term of the NAV and the benchmark. Returns are from 10,000 Monte Carlo simulations for a fund with median characteristics with the exception of age, which is set to 0 years, 7 years and 14 years.

PANEL A	Med	Med*	P (s.e.)	P* (s.e.)
α	0.0080	0.0090	0.0066	0.0068
			(0.0040)	(0.0044)
β_1	0.9251	0.6614	1.0062	0.8199
			(0.0789)	(0.0867)
β_2	-0.0162	0.1057	-0.2100	-0.0458
			(0.3215)	(0.3531)
R^2	0.1669	0.1205	0.5596	0.4119

PANEL B	Med	Med*	P (s.e.)	P* (s.e.)
α	0.0082	0.0095	0.0080	0.0067
			(0.0034)	(0.0038)
β_1	1.5403	1.7093	1.4082	1.3659
			(0.1073)	(0.1414)
β_2	-0.7939	-0.3694	-0.3117	-0.2137
			(0.2723)	(0.3603)
R^2	0.1620	0.1548	0.5951	0.4247

PANEL C	Med	Med*	P (s.e.)	P* (s.e.)
α	0.0095	0.0095	0.0077	0.0069
			(0.0034)	(0.0039)
β_1	1.5791	1.6983	1.3379	1.3480
			(0.1118)	(0.1424)
β_2	-0.6958	-0.4512	-0.2408	-0.1936
			(0.2716)	(0.3608)
β_3	0.1951	0.1828	1.0908	0.5876
			(0.5491)	(0.5762)
R^2			0.6073	0.4293

Table 5: Asset Management Firm Returns and Fundamentals

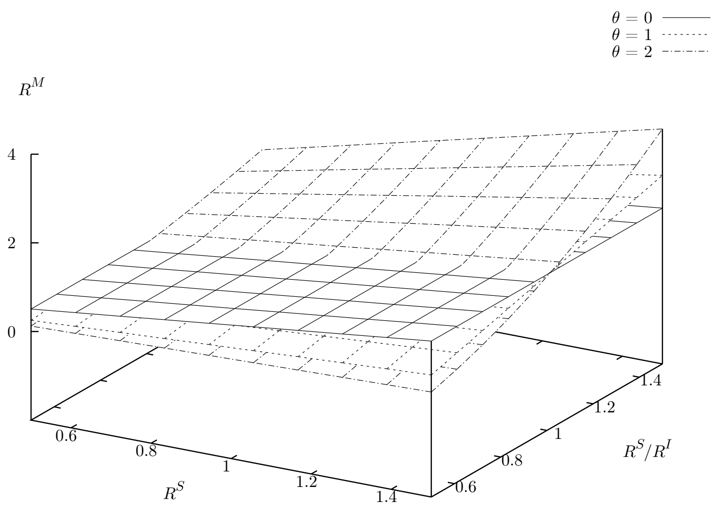
We estimate in panels A, B and C, respectively, the following three regressions

$$R_t^i = \alpha^i + \beta_1^i R^{sp,t} + \beta_2^i R^{leh,t} + \epsilon_t \qquad (49)$$

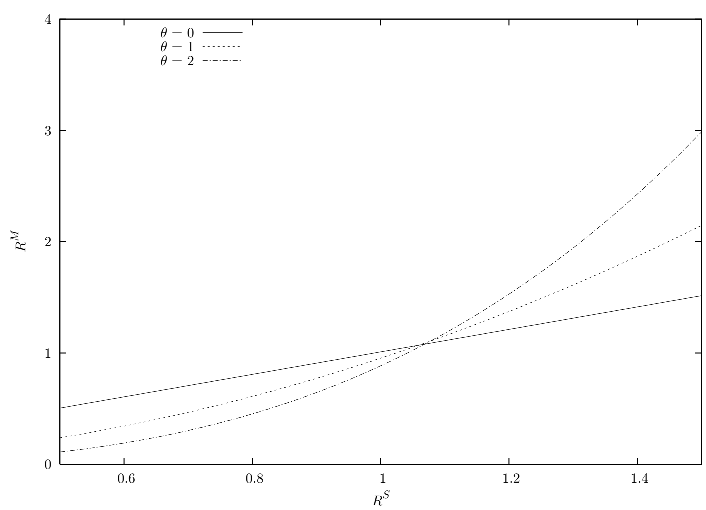
$$R_t^i = \alpha^i + \beta_1^i \Sigma_j w_t^j R_t^j + \beta_2^i \Sigma_j w_t^j (R_t^j R_{bench,t}^j) + \epsilon_t \qquad (50)$$

$$R_t^i = \alpha^i + \beta_1^i \Sigma_j w_t^j R_t^j + \beta_2^i \Sigma_j w_t^j (R_t^j R_{bench,t}^j) + \beta_3^i \Sigma_j w_t^j F_t^i + \epsilon_t \qquad (51)$$

where the median coefficients are reported for a sample of 27 firms and a narrow sample of 9 firms (denoted by *) with 100 or more monthly data points. The regressions are run quarterly overlapping. The portfolios are, correspondingly, industry portfolios with 27 or 9 firms. The weighted sums refer to AUM weights within a fund family. The benchmark is either the SP500 or Lehman index, as relevant. Flows, denoted by F, are total AUM growth less fund returns.

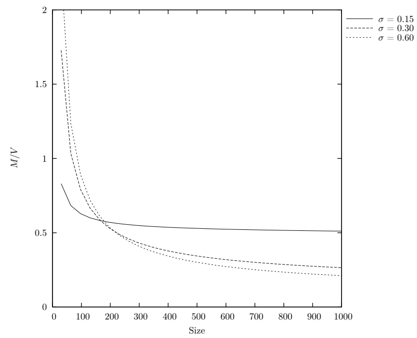


(a) R^M vs. R^S and R^S/R^I

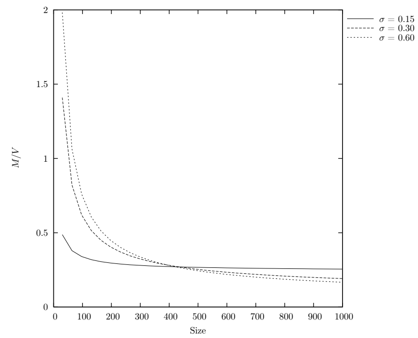


(b) R^M vs. R^S keeping $R^I = e^r = 1.0513$

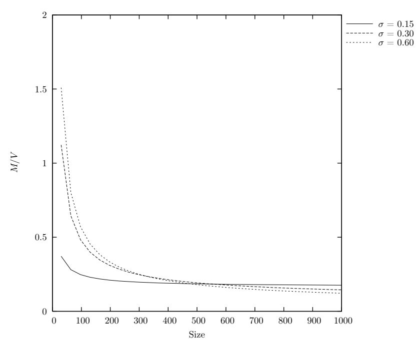
Figure 1: Expected return on fund's value versus NAV return and return on index
The figures show how the expected return on the fund's value varies with its NAV return and the return on the index, assuming fund flows in and out of the fund are governed by equation (7), with parameters $\sigma = 0.2, \sigma_I = 0.15$, $\rho = 075$, $r = 0.05$, $\nu = 0$.



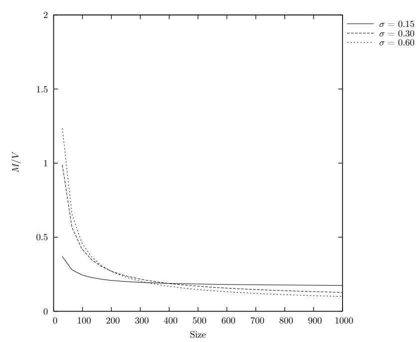
(a) Age = 2 years



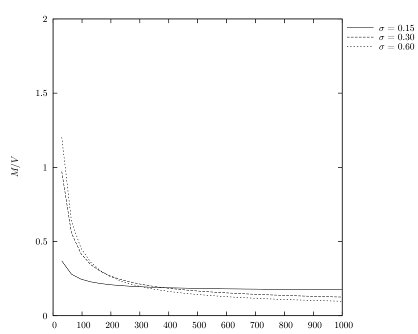
(b) Age = 7 years



(c) Age = 30 years



(d) Age = 100 years



(e) Age = 200 years

Figure 2: M/V versus size

The figure shows the fund's value as a fraction of NAV for different fund sizes, and different assumptions about the fund's initial age and volatility. All other parameters are assumed to take the median value for the appropriate fund type, and valuations are done using 10,000 Monte Carlo simulations.

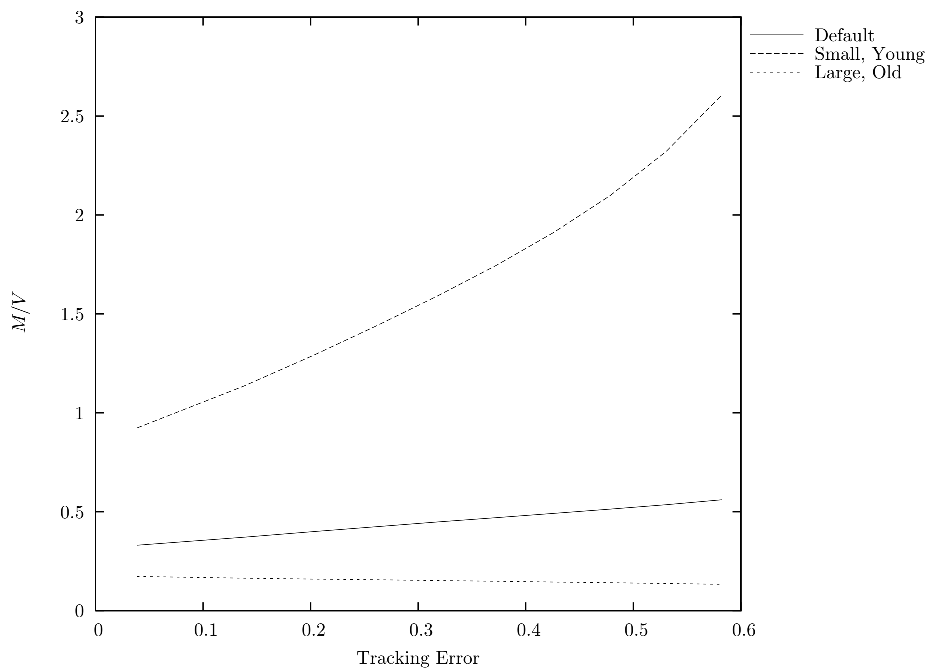


Figure 3: M/V versus Tracking Error Volatility

The figure shows the fund's value as a fraction of NAV for different assumptions about the tracking error volatility. All other parameters are assumed to take the median value for the appropriate fund type, and valuations are done using 100,000 Monte Carlo simulations.